32-cm Wavelength Radar Mapping of the Moon

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***Abstract*— We present our effort for producing a high-resolution 32-cm wavelength synthetic aperture radar map of the Moon using ground based measurements with the EISCAT UHF radar. We discuss coding, decoding, Doppler north-south ambiguity mitigation, focusing, and clock error mitigation. We also show preliminary results from a test measurement.**

I. Introduction

Polarimetric radar studies of the Moon are useful as they provide a way of probing the sub-surface geochemical properties and the rock abdundance of lunar regolith [1]. Previously such maps have been produced at wavelengths of 3.8 cm [2], 70 cm [1], [3] and 7.5 m [4]. To our knowledge, the 32 cm wavelength used by the EISCAT UHF system has not been previously used for such studies. Different wavelengths probe the surface at different depths. Echo intensity also tells of the surface roughness at the radar wavelength scales [4]. Thus a 32 cm map would provide valuable information to complement space probe measurements and other previously published high resolution Lunar maps.

Most high-resolution Earth based Lunar mapping efforts have been conducted with a very narrow transmitter beam, which allows the Doppler north-south ambiguity to be avoided by beam positioning. In other cases, interferometry has been used to solve the ambiguity [4]. The EISCAT UHF transmistter antenna has a beam full width at half maximum of approximately 0.5 degrees, which illuminates the whole Moon simultaneously. Thus, there will be mixing of the north and south of the apparent Doppler equator. However, it is still possible to position the first null of the beam in such a way that ambiguous echos from the other side of the Doppler equator are sufficiently attenuated.

II. Mapping procedure

Our process is similar to previous range-Doppler mappings [5] of the Moon with the exception that we use a long coded ≈ 2 ms pulse to compensate for the small antenna gain. Also, instead of correcting the mean Lunar Doppler shift in the transmission frequency, we perform the Doppler correction in software to the recorded raw voltage data. This allows us to perform Lunar measurements without any hardware modifications to the EISCAT system.

A. Pulse compression

The instantaneous Doppler spread of the Moon is between 10-15 Hz during a typical measurement. Thus, we can assume that the target is approximately stationary during the time when the ≤ 2 ms transmission pulse travels through the target. The only significant factor is the slowly changing mean Doppler shift ω_t between the Moon and the observer, which can be assumed constant during one inter-pulse period.

If we assume that each range is Doppler shifted uniformly by ω and that the complex valued $\zeta_r \in \mathbb{C}$ contains the phase and backscatter amplitude for each range, the measurement equation for a single Lunar echo is:

$$m_t = \xi_t + \sum_r \epsilon_{t-r}\zeta_r \exp\left(i\omega(t-r)\right). \tag{1}$$

We denote our transmission envelope with ϵ_t, and the complex Gaussian valued measurement errors with ξ_t. By writing

$$\exp(-i\omega r)\exp(i\omega t) = \exp(i\omega(t-r)), \tag{2}$$

and replacing $\zeta'_r = \zeta_r \exp(-i\omega r)$ we can write the equation as:

$$m_t = \xi_t + \sum_r \epsilon_{t-r}\zeta'_r \exp\left(i\omega t\right). \tag{3}$$

We now divide by $\exp(i\omega t)$, set $\xi'_t = \exp(-i\omega t)\,\xi_t$. We arrive at:

$$m_t \exp\left(-i\omega t\right) = \xi'_t + \sum_r \epsilon_{t-r}\zeta'_r, \tag{4}$$

which is the measurement equation of a coherent target with the exception that the measurement is multiplied by a complex sinusoid. This equation is valid for a single inter-pulse period.

To estimate ζ'_r, we first multiply our measurement with the Doppler correction $\exp(-i\omega t)$, and convolve the result with the inverse filter

$$\lambda_t = \mathcal{F}_D^{-1}\left\{\frac{1}{\mathcal{F}_D\epsilon_t}\right\}_t, \tag{5}$$

which is known to be the target backscatter amplitude maximum *a posteriori estimate* in the case of high SNR (i.e., a Wiener filter with the assumption of high SNR). This can also be understood as division by the transmission envelope in frequency domain for infinitely extended aperiodic signals (here $\mathcal{F}_D$ is an infinitely extended zero-padded discrete Fourier transform). If we examine the estimation error variance for ζ'_r, we see that it is the same as that of a stationary spread target as obtained in [7], [8]. We have used the same code optimality criterion here.

On a longer time-scale, the Lunar Doppler shift changes slowly, so we have to take into account the changing Doppler shift ω_t. This is done by making the Doppler correction term a slowly chirping complex sinusoid $\exp(-i\omega_t t)$, although the chirp rate during a single inter-pulse period is insignificant, and the frequency can be assumed constant.

After decoding, we obtain range and time (here t is the IPP index) dependent backscatter amplitude $\zeta_{r,t} + n_{r,t}$ with additional complex Gaussian noise $n_{r,t}$, which has a range dependent covariance structure from the inverse filtering step. The estimate of $\zeta_{r,t}$ can be then used to obtain an focused or unfocused range-Doppler map of the target. The unfocused map is obtained simply by making an independent power spectrum estimate for each range of $\zeta_{r,t}$.

In order to form the focused map of the target, we also have to take into account range and Doppler migration caused by Lunar libration during the integration period. We assume a spherical shape for the Moon, and with the help of the Lunar ephemeris, we calculate the corresponding Lunar coordinates and their Doppler shifts contributing to the backscatter at each range gate. This is then used to form a theory matrix A that describes the measured complex backscatter amplitude $\zeta_{r,t}$ in terms of the complex backscatter coefficients in Lunar coordinates $\sigma(x, y)$. To speed up computations, the resulting theory matrix A and error covariance matrix for $n_{r,t}$ can be assumed orthogonal ($A^H A \approx I$), so we can form our estimate by correlation $\hat{x} = A^H m$:

$$\sigma(x, y) = \frac{1}{T} \sum_{t=0}^{T} \zeta_{r(t,x,y),t} \exp\{i\omega_l(t, x, y)t\}, \qquad (6)$$

where $r(t, x, y)$ and $\omega_l(t, x, y)$ are the range gate and Doppler shift of Lunar coordinate (x, y) at time instant t.

A slightly better, but more time-consuming estimate can be obtained by using the full linear solution, with the covariance structure of $n_{r,t} \sim N(0, \Sigma)$:

$$\hat{x} = (A^H \Sigma^{-1} A)^{-1} A^H \Sigma^{-1} m, \qquad (7)$$

where vector $\hat{x}$ contains parameters $\sigma(x, y)$ and vector m contains the measurements $\zeta_{r,t}$ in such a way that the forward theory matrix A describes the measurements in terms of the parameters $m = Ax$.

III. Clock errors

For a point-like feature in a range-Doppler image, the signal can be written as:

$$s(t) = \exp(2\pi\omega i t) \exp(2\pi\omega\sigma_a i t), \qquad (8)$$

where σ_a is the Allan deviation of the clock. Here $\exp(2\pi\omega\sigma_a i t)$ is the error-term, ω is the radar frequency and t is time. Heuristically, one can say that once the clock error term is off by more than $\pi/4$, our coherence is lost. Using this definition, we can define a average coherence time achievable with the clock:

$$\tau = (8\omega\sigma_a)^{-1} \qquad (9)$$

To our knowledge, the best Rb clocks available at the moment have a stability of $\sigma_a = 4 \cdot 10^{-13}$ (100 s time scale). This gives us a coherence time of 340 s. On the other hand, a typical active hydrogen maser clock has an Allance deviation of $\sigma_a = 7 \cdot 10^{-15}$ (one hour time scale), which gives us a coherence time of 19300 s. As we only have a Rb clock, we have to be able to correct the clock drift in some way if we want to achieve better than 340 s integration periods.

The clock stability directly defines the Doppler resolution. If we assume that the Moon has a Doppler spread of ω_M, and the radius of the Moon is 1738 km, our best achievable resolution in meters near the Doppler north pole is approximately

$$\Delta r = \frac{2 \cdot 1738 \cdot 10^3}{\tau\omega_M} \text{m}. \qquad (10)$$

For $\omega_M = 10$ Hz and $\tau = 340$ s, we get a resolution of $\Delta r = 1022$ m. In the case of a hydrogen maser, we get $\Delta r = 18$ m.

A. Clock error recovery

In synthetic aperture radar mapping, clock errors result in smearing of the image in Doppler direction. As we are using GPS stabilized Rb clocks, we expect the clock to drift several radar wavelengths during an hour. It would be nice if there would be a way to correct clock drifts from the Lunar measurement itself.

For a point-like target it can be shown that clock errors can be recovered from the data. Assuming that the target has a certain range with a known point Doppler shift $\phi \in \mathbb{R}$ and assuming no measurement errors, the measurement of that range gate can be stated as

$$m(t) = a \exp\{i\phi(t + \varepsilon(t))\}, \qquad (11)$$

where $m(t)$ is the measurement, $a \in \mathbb{C}$ is the unknown backscatter coefficient containing both phase and magnitude, $t \in \mathbb{R}$ is time, and $\varepsilon(t) \in \mathbb{R}$ is the clock error, which we assume to be a zero-mean stochastic process, i.e., $\forall t, \mathrm{E}\,\varepsilon(t) = 0$. We arrange the terms as follows

$$-i\phi^{-1} \log m(t) - t = \varepsilon(t) - i\phi^{-1} \log a. \qquad (12)$$

In this case, we don't need to estimate a, so we can solve $\varepsilon(t)$ making use of the fact that $\int_0^\infty i\phi^{-1} \log m(t) + t dt = i\phi^{-1} \log a$, and get

$$\varepsilon(t) = -\frac{i \log m(t)}{\phi} - t + \int_0^\infty \frac{i \log m(t)}{\phi} + t dt + \frac{2\pi n}{\phi}, \qquad (13)$$

where $n \in \mathbb{Z}$. Thus, it is in theory possible to recover clock errors from noise-free data.

The Lunar map also contains similar, albeit not exactly the same, types of features. E.g., the leading edge of the Moon is nearly a point-target. And sharp shadows formed at the edges of craters are analguous. In practice, it should be possible to use some sort of prior information that promotes sharp features in the image. An example of such a prior is the total variation prior [6].

IV. Test run

To test the feasibilty of the EISCAT system for Lunar studies, we have conducted several test experiments during November 2008. We used the EISCAT Tromso UHF system in monostatic mode at full 2 MW peak power. The coded transmission pulses were 200-1825 μs long with bauds between 1-10 μs. The duty cycle was between 2-11%, with interpulse intervals carefully selected so that the Lunar echos would fit between transmission slots during the whole experiment. The ephemeris was obtained from the NASA JPL Horizon's system.

We sampled our data at 4 MHz using a Universal Software Radio Peripheral (USRP[1]) and stored the 16-bits per sample raw voltage data to disk in baseband – all processing was done off-line to this data, so no modifications to the EISCAT system were required. We also recorded the transmission envelope from the waveguide and used it for decoding in the off-line processing stage to mitigate decoding errors caused by the non-ideal transmission waveform.

A part of a raw unfocused delay-Doppler image using 2 μs baud-length is shown in Fig. 1. The image is taken over a 600 s integration period. This is close to the limits of the capabilities of our Rb clock. The resolution is approximately 600 m in range and Doppler direction, although Doppler smearing is already expected with such a long integration time.

We used several different transmission codes with baud-lengths ranging from 1 μs to 10 μs. The transmission codes were Kroenecker product (i.e., sub-pulse coded) codes derived from the 13-bit Barker code or an optimal sub-sequence of such a code. The optimization criteria was the posterior estimation variance of stationary spread target backscatter amplitude [7], [8].

V. Future Work

We have demonstrated that the feasibility of creating a high-resolution 32-cm Lunar map with the EISCAT UHF radar. We plan to continue the work to:

- Produce a full focused map using both same and opposite sense circular polarization
- Compare results to other measurements at different wavelengths
- Investigate the possibility of long baseline interferometric measurements using the EISCAT system

VI. Conclusions

We have outlined our ongoing work to produce a high-resolution 32 cm focused polarized synthetic aperture radar map of the Moon. The main differences to previous work is that we are using coded long pulses and pointing the beam pattern nulls to produce an unambiguous range-Doppler map. We also have several ideas for correcting the clock errors to increase the coherent integration time further.

We have performed preliminary measurements to prove that a high-resolution map is feasible using EISCAT. Our goal is to proceed to measure a full focused Lunar map with same- and opposite-circular polarizations.

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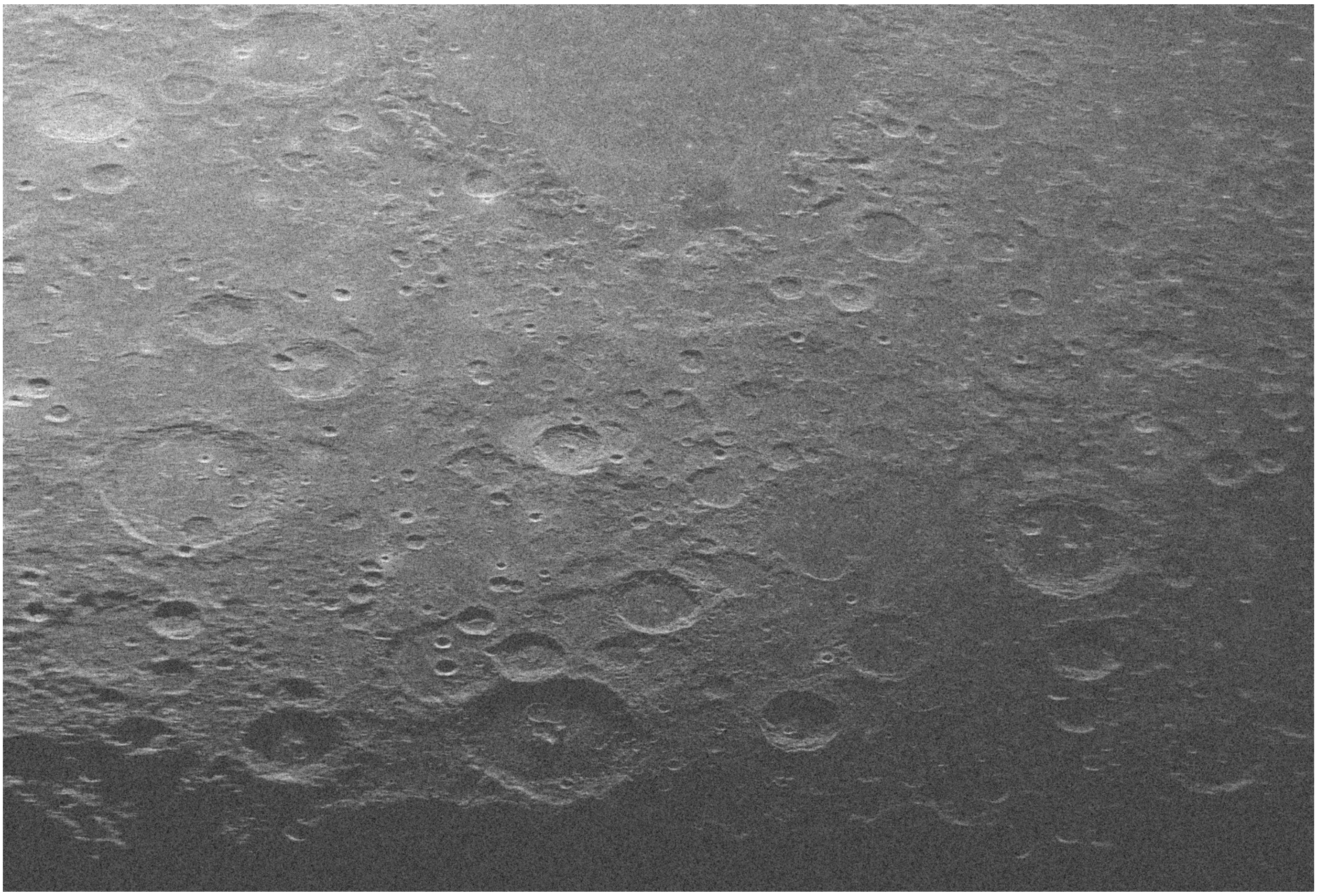

Fig. 1. An opposite sense (quasispecular or coherent scatter) circular polarized unfocused delay-Doppler image obtained with the EISCAT 926 MHz UHF system. The range resolution is approximately 600 m. Because the image is still unfocused, Doppler smearing caused by the changing range rate can be seen on the edges.