Sodankylä Geophysical Observatory

Reports

15th Inverse Days 2009

at Luosto

Sodankylä, Finland

16th – 18th December 2009

Abstracts

Edited by Matti Vallinkoski

Report No 59

Sodankylä 2009

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UNIVERSITY of OULU
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Sodankylä 2009

15th Inverse Days

16th - 18th December 2009

Amethyst Spa, Luosto, Lapland

Photo Quiz: Which photo does not belong to the same category? Note the existence and uniqueness of the answer.

Welcome to Luosto, Lapland!

The annual Inverse Days of the Finnish Inverse Problems Society are organized in the Luosto ski resort in Central Lapland. The local organizing committee comes from the Sodankylä Geophysical Observatory.
As usual, the Inverse Days consist of talks, good food and drinks, sauna, swimming (hole in the ice, outside hot tub, spa), conference dinner and high ambition on great science + walrus wrestling.

Conference Hotel and Accommodation

The conference is organized in the Luostotunturi hotel also known as Amethyst Spa. All talks will be given in the hotel auditorium on the first floor. Lunches and coffee as well as the conference dinner will be served in the hotel. The conference sauna will be on Wednesday 500 metres from the hotel on the shore of a small lake with a magnificent view over the Luosto fell.
Accommodation in hotel rooms or in nearby log cabins.

Transportation

One can access Luosto via the Rovaniemi railway station or the Rovaniemi airport. There are frequent coaches to Luosto. The organizers will arrange a bus transportation (by Haapasen Liikenne) on Wednesday 16th morning from Rovaniemi to Luosto, and on Friday 18th afternoon back to Rovaniemi from Luosto. If you come by car, please use your gps or map.

Talks

Talks will be held in the hotel auditorium. Every talk has a 30-minute slot consisting of:

20	Minutes for the talk
5	Minutes for questions, comments and discussion
5	Minutes pause.

Exceeding the time limits shall lead to a severe penalization by the session chair.
Standard projectors, chalk and a flip board will be available as well as an audio system.

Posters

There will be a designated area for posters. One does not need to send poster abstracts. There will be no official poster session, however, the posters will be on the walls for the entire duration of the conference.

Wednesday 16th December 2009

08:10	Bus transportation from the Rovaniemi railway station
08:15	Stop at the Scandic Hotel in Rovaniemi
09:15	Bus transportation from the Rovaniemi airport to Luosto
11:00	Registration + Lunch + Check-in to the hotel and log cabins

Afternoon Session 1, Chair Matti Lassas

12:20	Markku Lehtinen	Opening of the Inverse Days
	Lassi Roininen	Discussion of practical issues
12:30	Kirsi Peltonen	Heisenberg group-connecting geometries in dimensions 2,3 and 4
13:00	Patricia Gaitan	Simultaneous Identification of the Diffusion Coefficient and the Potential for the Schrödinger Operator with only one Observation
13:30	Roberta Bosi	Quantum kinetic equations: direct and inverse problems
14:00	Oana Constantinescu	The inverse problem of the calculus of variations
14:30	Coffee break	

Afternoon Session 2, Chair Samuli Siltanen

15:00	Harri Auvinen	The variational Kalman filter and an efficient implementation using limited memory BFGS
15:30	Juha-Matti Perkkiö	Ray transform in Finsler spaces
16:00	Michel Cristofol	Stability Reconstruction of Two Coefficients Simultaneously in a Nonlinear Parabolic Equation
16:30	Sampsa Pursiainen	Conditionally Gaussian Hypermodels for Cerebral Source Localization

Evening Sessions

17:30	Annual meeting of the Finnish Inverse Problems Society
19:00	Conference sauna and dinner at Metsä-Luosto (500 metres from the hotel)

Thursday 17th December 2009

Morning Session, Chair Lassi Päivärinta

09:00	Matti Lassas	Inverse problems for wave equation with new type of measurements
09:30	Sari Lasanen	Posterior convergence for correlation priors
10:00	Juha Vierinen	Moving point-target detection using Bayesian model selection
10:30	Coffee break	
11:00	Chris Johnson	Finite-Element Discretization Strategies for the Electrocardiographic (ECG) Inverse Problem
11:30	Samuli Siltanen	Computerized inversion for spoken language

Afternoon Session, Chair Marko Laine

12:00	Esa Turunen	EISCAT3D - A European Three-Dimensionally Imaging Radar for Atmospheric and Geospace Research
12:30	Marko Vauhkonen	From inverse problems to practical applications
13:00	Lunch	
14:00	Mikko Orispää	Developments in FLIPS
14:30	Antti Solonen	Simulation-Based Optimal Design

Parallel Session

14:00 SAB meeting and/or Random things for random people

Evening Session

19:00 Conference Dinner at hotel

Friday 18th December 2009

Morning Session, chair Valeriy Serov

09:00	Matias Dahl	Descending maps between slashed tangent bundles
09:30	Ioan Bucataru	Helmholtz conditions for projective metrizability problem and their formal integrability
10:00	Lauri Harhanen	Convex source support in half-plane
10:30	Nuutti Hyvönen	Inverse backscattering in electric impedance tomography
11:00	Lunch + Check-out	

Afternoon Session, chair Mikko Kaasalainen

12:00	Olivier Poisson	Probing for inclusions in heat-conductive bodies
12:30	Bastian Harrach	Exact shape reconstruction by one-step linearization in electrical impedance tomography
13:00	Tanja Tarvainen	Approximation error approach for compensating modelling errors between the radiative transfer equation and diffusion approximation in diffuse optical tomography
13:30	Petri Hiltunen	A combined reconstruction-classification algorithm for diffuse optical tomography
14:00	Aku Seppänen	Electrical impedance tomography imaging of concrete
14:30	Professors	Closure of the Inverse Days + Pertti 'Lande' Lindfors Prize

Departure

15:15 Departure by bus for the evening flight to Helsinki + night trains

Abstracts

Heisenberg group-connecting geometries in dimensions 2,3 and 4

Kirsi Peltonen

Department of Mathematics, Helsinki University of Technology
FIN-02015 Espoo, Finland
kirsi.peltonen@tkk.fi

We report on recent developments on a beautiful connection related to planar complex analysis, real conformal geometry and complex hyperbolic geometry. Planar Beltrami equation has a natural counterpart in the Heisenberg group when equipped with the sub-Riemannian structure. We give a general overview and invitation to study related structures. Especially we show how invariant non-standard CR-structures arise in this setting and provide interesting backround for applications.

Simultaneous Identification of the Diffusion Coefficient and the Potential for the Schrödinger Operator with only one Observation

Patricia Gaitan

Aix-Marseille University
13625 Aix-en-Provence
patricia.gaitan@univmed.fr

Let $\Omega = \mathbb{R} \times (0, d)$ be an unbounded strip of $\mathbb{R}^2$ with a fixed width d. We consider the following Schrödinger equation

$$\begin{cases} i\partial_t q + a\Delta q + bq = 0 \text{ in } \Omega \times (0, T), \\ q(x, t) = F(x, t) \text{ on } \partial\Omega \times (0, T), \\ q(x, 0) = q_0(x) \text{ in } \Omega. \end{cases}$$

We simultaneously determine with only one observation two independent coefficients: the diffusion coefficient a and the potential b from the measurement of $\partial_\nu(\partial_t^2 q)$ on a part Γ^+ of the boundary.
Our main result is

$$\|a - \widetilde{a}\|^2_{L^2(\Omega)} + \|b - \widetilde{b}\|^2_{L^2(\Omega)} \le C\Bigg(\|\partial_\nu(\partial_t^2 q) - \partial_\nu(\partial_t^2 \widetilde{q})\|^2_{L^2((-T,T)\times\Gamma^+)} + \sum_{i=0}^{2} \|\partial_t^i (q - \widetilde{q})(\cdot, 0)\|^2_{H^2(\Omega)} \Bigg),$$

where $C = C(\Omega, \Gamma, T)$.

Quantum kinetic equations: direct and inverse problems

Roberta Bosi

Department of Mathematics (UCL)
Gower street, WC1E 6BT London, UK
rbosi@math.ucl.ac.uk

The kinetic quantum transport equations describe the evolution of an ensemble of charged quantum particles in the phase space, according to the Wigner formalism. The presentation focusses on the derivation of such equations from the Schrödinger and the density matrix formalism even in the nonlinear case and in presence of dispersive terms (e.g. quantum Fokker-Planck systems).

Some related direct problems will be discussed, such as existence and classical limit, and corresponding inverse problems will also be mentioned (e.g. determination of the doping profile in semiconductor devices).

Semi-basic 1-forms and the inverse problem for time-dependent second-order ordinary differential equations

Ioan Bucataru and Oana Constantinescu

Al.I.Cuza University of Iasi, Faculty of Mathematics
Romania
oanacon@uaic.ro

We present a reformulation of the inverse problem for time dependent SODE in terms of semi-basic 1-forms, using only the Frölicher-Nijenhuis theory on the first jet bundle. We seek for solutions of the inverse problem on two classes of semi-basic 1-forms. The first class consists of Poincaré-Cartan 1-forms of some Lagrangian functions. The second class consists of semi-basic 1-forms that are equivalent with Poincaré-Cartan 1-forms of some Lagrangian functions. Both classes of semi-basic 1-forms lead to the classic formulation of Helmholtz conditions in terms of matrix multipliers. The second class provides dual symmetries and first integrals for the given SODE. (arXiv:0908.1631)

The variational Kalman filter and an efficient implementation using limited-memory BFGS

Harri Auvinen

Lappeenranta University of Technology
Finland
Harri.Auvinen@lut.fi

In the field of state space estimation and data assimilation, the Kalman filter (KF) and the extended Kalman filter (EKF) are among the most reliable methods used. However, KF and EKF require the storage of, and operations with, matrices of size $n \times n$, where n is the size of the state space. Furthermore, both methods include inversion operations for $m \times m$ matrices, where m is the size of the observation space. Thus Kalman filter methods become impractical as the dimension of the system increases. We introduce a Variational Kalman filter (VKF) method to provide a low storage, and computationally efficient, approximation of the KF and EKF methods. Furthermore, we introduce a Variational Kalman smoother (VKS) method to approximate the fixed-lag Kalman smoother (FLKS) method. Instead of using the KF formulae, we solve the underlying maximum a posteriori optimization problem using the limited-memory BFGS (LBFGS) method. Moreover, the LBFGS optimization method is used to obtain a low-storage approximation of state estimate covariances and prediction error covariances. The simulated assimilation results are presented and compared with KF and EKF, respectively.

Ray transform in Finsler-spaces

Juha-Matti Perkkiö

Helsinki University of Technology
Finland
jperkkio@cc.hut.fi

For an open non-trapping Finsler-space (M,F) one can define the manifold of geodesics $GM = SM/_{\Phi_H}$ as the quotient space where the unit sphere bundle $SM \subset TM$ is divided by the geodesic flow $\Phi_H : \mathbf{R} \times SM \to SM$. Then one defines the (anisotropic) geodesic ray transform as

$$I : C_c^\infty(SM) \to C_c^\infty(GM) \quad ; \quad If(\gamma) = \int_{-\infty}^{\infty} f(\gamma(t), \dot\gamma(t))dt.$$

Clearly $I(Hf) = 0$ for the geodesic spray H on SM, but one may ask for which subspaces $W \subset C_c^\infty(SM)$ the mapping $I|_W : W \to C_c^\infty(GM)$ is injective. We provide some preliminary results for Finsler-spaces of non-positive curvature and for Finsler-spaces without conjugate points.

Stability Reconstruction of Two Coefficients Simultaneously in a Nonlinear Parabolic Equation

Michel Cristofol

Université d'Aix-Marseille
France
michel.cristofol@univ-cezanne.fr

We consider the following nonlinear parabolic "KPP type" equation which modelizes population dynamics in heterogeneous media :

$$\begin{cases} \partial_t u_\gamma = D\Delta u_\gamma + u_\gamma(\mu(x) - \gamma(x)u_\gamma) \text{ in } Q, \\ u_\gamma(t,x) = 0 \text{ on } \Sigma, \\ u_\gamma(0,x) = u_i(x) \text{ in } \Omega, \end{cases} \qquad (P_{\mu,\gamma}) \tag{1}$$

where u_γ corresponds to the population density , Ω is a bounded subdomain of $\mathbb{R}^d$, of smooth (C^2) boundary $\partial\Omega$, $Q := (0,+\infty) \times \Omega$ and $\Sigma := (0,+\infty) \times \partial\Omega$. We deal with the problem of forecasting the regions at higher risk for newly introduced invasive species. Favourable and unfavourable regions may indeed not be known a priori, especially for exotic species, whose hosts in native range and newly-colonized regions can be different. Assuming that the species dynamics are governed by a logistic-like reaction-diffusion equation, we consider successively two problems

- determine and reconstruct the intrinsic growth rate $\mu(x)$ (see [1])
- determine and reconstruct simultaneously the intrinsic growth rate $\mu(x)$ and the intraspecific competition coefficient $\gamma(x)$ (see [2]), these reconstructions being obtained with as less as possible of data.

The main results can now be stated:

- for some times (t_0, t_1), and any subset $\omega \subset\subset \Omega_1$, it is possible to estimate the function $\mu(x)$ in Ω, basing ourselves only on measurements of $u(t,x)$ over $(t_0,t_1) \times \omega$, and on a single measurement of $u(t,x)$ in the whole domain Ω at a time $T' = \frac{t_0+t_1}{2}$. We do not need the knowledge of $\gamma(x)$.
- for some times (t_0, t_1), and any subset $\omega \subset\subset \Omega_1$, it is possible to estimate the functions $\mu(x)$ and $\gamma(x)$ in Ω, basing ourselves only on measurements of $u(t,x)$ over $(t_0-\delta, t_1+\delta)\times\omega$ for any $\delta \in (0,t_0)$, and on two measurements of $u(t,x)$ in the whole domain Ω at times t_0 and t_1.

I will speak essentially about the most recents results concerning the reconstruction of two coefficients. The proofs of these results bear on different Carleman-type estimate, the maximum principle and Hopf's lemma.

References

1. M Cristofol and L Roques, A Biological invasions: "Deriving the regions at risk from partial measurements." *Mathematical Biosciences* 215(2):158–166, 2008.
2. M Cristofol and L Roques, "Simultaneous reconstruction of two coefficients in a nonlinear parabolic equation " *submitted to CRAS*. 2009.

Conditionally Gaussian Hypermodels for Cerebral Source Localization

Sampsa Pursiainen

Institute of Mathematics, Helsinki Univ. of Technology
P.O.Box 1100 FI-02015, Finland
sampsa.pursiainen@tkk.f

This talk deals with EEG/MEG imaging of the brain. EEG/MEG imaging can be modeled as a linear or nonlinear inverse problem where the unknown parameter is the primary source current density caused by the neural activity in the brain. In inverse problems, prior information is often qualitative in nature, making the translation of the available information into a computational model a challenging task. This study proposes a generalized gamma family of hyperpriors allowing the primary current density to be focal and also advocates a fast and efficient iterative algorithm, the Iterative Alternating Sequential (IAS) algorithm for computation of MAP estimates. It is shown that for particular choices of the scalar parameters specifying the hyperprior, the algorithm effectively approximates popular regularization strategies. The posterior densities are explored by means of a Markov Chain Monte Carlo (MCMC) strategy suitable for this family of hypermodels. The computed experiments suggest that the known preference of regularization methods for superficial sources over deep sources is a property of the MAP estimators only, and that estimation of the posterior mean in the implemented model is better adapted for localizing deep sources.

Inverse problems for wave equation with new types of measurements

Matti Lassas

University of Helsinki
Finland
Matti.Lassas@helsinki.fi

We discuss recent new methods to solve inverse problems for the wave equation. We will consider modified time-reversal methods, which are a combination of the boundary control method and an iterative time-reversal scheme. These lead to adaptive imaging of coefficient functions of the wave equation using focussing waves in an unknown medium. We also discuss other new measurement settings where measurements which we can do are very limited or poorly known.

Posterior convergence for correlation priors

Sari Lasanen

University of Oulu
Finland
sari.lasanen@oulu.fi

Correlation priors X_t, $t \geq 0$, are certain weakly stationary zero-mean Gaussian smoothness priors. Their prior information is contained in the correlation function $C(t-s) = \mathbf{E}[X_t X_s]$ which describes our beliefs about the unknown function. The main benefits of the correlation priors when compared to ordinary smoothness priors are the possibility to choose the correlation length and spatial homogeneity up to the boundary. The main disadvantage is that using the matrix $C_{ij} = C(t_i - t_j)$ in numerical calculations is slower than using ordinary smoothness priors. This problem has been approached by Roininen et al. who introduced finite-dimensional approximations of correlation priors. In practice, these approximations allow as efficient calculations as in the case of ordinary smoothness priors. Roininen et al. have demonstrated that approximative correlations converge to actual correlation functions. In this presentation, the implications of this result for posterior convergence are considered.

Moving point-target detection using Bayesian model selection

Juha Vierinen

Sodankylä Geophysical Observatory
Finland
j@sgo.fi

We investigate the problem of identifying multiple moving point-targets from a large set of noisy measurements with the framework of Bayesian model selection. We describe an iteration that can be used to approximate the peak of the posteriori probability density. We demonstrate the method with high power large aperture radar measurements of meteor head echos and space debris and find that the method increases detection rates compared to previously used methods. With small modifications, the method should be applicable to other types of moving target detection problems.

Finite-Element Discretization Strategies for the Electrocardiographic (ECG) Inverse Problem

Chris Johnson

University of Utah/SCI Institute
USA
crj@sci.utah.edu

Successful employment of numerical techniques for the forward and inverse electrocardiographic (ECG) problems requires the ability to both quantify and minimize approximation errors introduced as part of the discretization process. Conventional finite-element discretization and refinement strategies effective for the forward problem may become inappropriate for the inverse problem because of the latter's ill-posed nature. Such an observation leads us to develop discretization strategies specifically for the inverse ECG problem. Quantitatively analyzing the connection between the ill-posedness of the continuum inverse problem and the ill-conditioning of its discretized version, we propose strategies involving hybrid-shaped finite elements so as to optimize the discretization of the inverse ECG problem. We also propose the criteria for evaluating the quality of the resulting discrete system. The efficacy of the strategies are demonstrated on a realistic torso model.

Computerized inversion for spoken language

SAMULI SILTANEN

University of Helsinki
Finland
samuli.siltanen@helsinki.fi

This talk describes a project that just got 1.2 million euros of funding from the Academy of Finland for the period 2010-2013.

The project aims at developing advanced computational machinery for the modelling and understanding of the production and perception processes of spoken language. The project involves research in four areas: (1) robust recognition of spoken language in natural environments, (2) high quality speech synthesis including affective factors, (3) computational modelling of brain responses evoked by speech sounds, and (4) non-invasive measurements of occupational vocal loading. By revolutionizing the use of computational inversion methods in all these areas, the project aims at considerable advances in the science of speech communication and at improved speech technology.

The consortium is a new, truly interdisciplinary collaboration between two groups of Finnish scholars; spoken language researchers and scientists in inverse problems. Methodologically, the study focusses on three fundamental computational inverse problems in spoken language research: (a) recovering the acoustic voice source and vocal tract characteristics from recorded speech, (b) modelling speech-evoked brain activity from electric and magnetic signals measured outside the head, and (c) imaging vocal folds non-invasively during speech. Research in all three problems is closely interconnected between the consortium partners and requires new, specialized algorithms for the computational solutions of the underlying spoken language problems. The new computerized inversion methods rely on complementing indirect measurement data by a priori expert knowledge. Since the data sets involved are very large, the essential difficulty in the project is to implement computationally effective yet accurate inversion algorithms.

The project has societal impact through the development of new technology, such as speech synthesizers capable of expressing emotions, speech recognizers that work in noisy surroundings, and voice load dosimeters that can give advance warning to professional voice users (e.g., teachers) about impending sick leaves due to loss of voice.

EISCAT3D - A European Three-Dimensionally Imaging Radar for Atmospheric and Geospace Research

Esa Turunen

EISCAT Scientific Association
P. O. Box 812, SE-981 28 Kiruna, Sweden
esa.turunen@eiscat.se

The EISCAT Scientific Association operates three high-power incoherent scatter radars, two advanced ionospheric sounders and a high-power ionospheric heating facility for active experiments in Northern Fennoscandia and Svalbard. A new project proposal called EISCAT3D has been accepted on the EU Roadmap of large research infrastructures of strategic importance for the next 20-30 years. The project aims at building a new-generation atmospheric, ionospheric and magnetospheric radar facility, with multiple measurement sites in three countries in Northern Fennoscandia. The radar would be capable of spatiotemporal imaging in $R^3 \times T$ instead of the current system's $R \times T$. The Sodankylä inverse group is leading the design of the new optimal hardware and software solution. Mathematically speaking, we need to model the scattering processes with stochastic Itò measures, the analysis with statistical inversion. New coding and sampling principles have to be made and understanding the noise behaviour in high signal-to-noise ratio situations has to be achieved. The data rates will be high, meaning petabytes per second that need to be analyzed and stored. This demands high-performance computation and networking. The project is challenging mathematically and technically. The construction phase will begin after the year 2015. We try to have a better understanding of the physics of the whole atmosphere and near-Earth space with radar measurements, and try to give at least partial answers to the question: How does the Solar-Terrestrial interaction in the coupled atmospheric system affect the climate of the Earth?

From inverse problems to practical applications

Marko Vauhkonen and Anssi Lehikoinen

University of Kuopio, Department of Physics and Numcore Ltd.
Kuopio, Finland
Marko.Vauhkonen@uef.fi

Many potential applications of inverse problems have arosen during the last decades. One important field includes diffuse imaging modalities such as optical tomography, capacitance tomography, magnetic induction tomography and electrical impedance tomography. Many potential applications of these techniques can be found for example in medical and industrial fields. In this talk we show some practical examples of utilizing electrical impedance tomography (EIT) in industrial field. Examples are mainly from application of EIT in pulp & paper and mining industries.

Developments in FLIPS

Mikko Orispää

Sodankylä Geophysical Observatory, Oulu Unit
University of Oulu
mikko.orispaa@oulu.fi

The Fortran Linear Inverse Problem Solver (FLIPS) is a Fortran95 module for solving large overdetermined linear statistical inverse problems. We present the recent developments in FLIPS achieved after the initial introductory talk given in Inverse Days 2006 in Tampere. These include, for example, the ability to delete data that has already been fed into the solver and the considerable performance increases for certain types of stochastic linear inverse problems. As an example, we present a fast method for smoothing riometer data using the data deletion feature.

Simulation-Based Optimal Design Using a Response Variance Criterion

Antti Solonen

Lappeenranta University of Technology
Finland
antti.solonen@lut.fi

Classical optimal design criteria suffer from two major flaws. First, they are routinely used in nonlinear problems by linearizing the model around a point estimate. Second, classical design methods are unavailable in ill-posed estimation problems, where the data does not contain enough information to identify all parameters. Bayesian optimal design can, in principle, solve these problems. A wide usage of Bayesian design methods is hindered by the lack of efficient and robust implementations that allow routine application in practical problems.

We point out a concrete recipe for implementing Bayesian optimal design. The approach utilizes an existing sample from the parameter posterior, nowadays routinely produced by efficient adaptive MCMC samplers. The proposed design criterion is based on searching for designs where the variance in model response is large (where a new measurement is able to discriminate different model parameters). The numerical sampling approach allows one to start the optimization of experiments at an early stage, before the estimation problem becomes well-posed and classical methods start to work. This potentially results in large reduction in the number of experiments needed to obtain a desired level of accuracy in the parameter estimates.

Descending maps between slashed tangent bundles

Ioan Bucataru and Matias Dahl

University College London, UK
matias.dahl@tkk.fi

Suppose M is a manifold without boundary. If F is a smooth map $F\colon TM \setminus \{0\} \to TM \setminus \{0\}$ in the slashed tangent bundle $TM \setminus \{0\}$, we study when F can be written as $F = (D\phi)|_{TM\setminus\{0\}}$ for a smooth map $\phi\colon M \to M$. In this case we say that F *descends*. The following theorem gives a differential-topological characterization of descending diffeomorphisms.

Theorem *If F is a diffeomorphism $F\colon TM \setminus \{0\} \to TM \setminus \{0\}$, then the following are equivalent:*

(i) *F be be written as $F = (D\phi)|_{TM\setminus\{0\}}$ for a smooth map $\phi\colon M \to M$.*
(ii) *$DF = \kappa_2 \circ DF \circ \kappa_2$, where $\kappa_2\colon TTM \to TTM$ is the canonical involution of the second iterated tangent bundle TTM, and DF is the tangent map of F.*

In this talk we also discuss geometric reformulations of condition *(ii)* when M is equipped with two Riemann metrics. In this case we obtain sufficient conditions for F to descend into an isometry $\phi\colon M \to M$. The motivation for deriving such conditions comes inverse problems in anisotropic media where media can only be reconstructed up to an isometry.

Helmholtz conditions for projective metrizability problem and their formal integrability

Ioan Bucataru

Al.I.Cuza University Iasi
Romania
bucataru@uaic.ro

In a joint work with Matias Dahl (see 1), we use the Frolicher-Nijenhuis theory to formulate four global Helmholtz conditions for the inverse problem of the calculus of variations in terms of regular partial differential operators. Within this general setting we show that the solutions of a system of second-order differential equations are the unparametrized geodesics of a Finsler metric (we say that the system of SODE is projectively metrizable) if and only if only two Helmholtz conditions are satisfied. In a joint project with Zoltan Muzsnay (2), we use Spencer theory to discuss the formal integrabillity of the corresponding two PDO. We prove that the only obstruction for the projective metrizability problem is due to the curvature of the system of SODE.

References

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2. Grifone, J., Muznay, Z., Variational principles for second order differential equations. Application of the Spencer theory to characterize variational sprays, World Scientific (2000).

Convex source support in half-plane

Lauri Harhanen

Department of Mathematics and Systems Analysis
Helsinki University of Technology
Finland
lauri.harhanen@tkk.fi

It is well known that the source term F in the Poisson equation

$$\Delta u = F \text{ in } \Omega, \quad \frac{\partial u}{\partial \nu} = 0 \text{ in } \partial\Omega,$$

cannot be uniquely determined from the boundary values of the solution u. The typical method for circumventing this problem is to consider only some specific class of sources, e.g., monopoles, dipoles, or characteristic functions. We, however, take an alternative approach and try to extract as much information as possible on the location of an arbitrary source. Although the support of F cannot be deduced from $u|_{\partial\Omega}$, it is possible to determine the smallest convex set that supports a source that produces the measured data on $\partial\Omega$. This set is referred to as the convex source support. In particular, the convex source support is contained in the convex hull of the support of F.

This work extends the concept of convex source support to the framework of inverse source problems for the Poisson equation in an insulated half-plane. We modify a previously introduced method for reconstructing the convex source support in bounded domains to our unbounded setting. The performance of the resulting numerical algorithm is demonstrated both for the inverse source problem and electrical impedance tomography with a single pair of boundary current and potential as the measurement data.

Inverse backscattering in electric impedance tomography

Nuutti Hyvönen

Department of Mathematics and Systems Analysis, Helsinki University of Technology
P.O.Box 1100, FI-02015 TKK, Finland
nhyvonen@math.tkk.fi

We consider (two-dimensional) electric impedance tomography with very sparse data that resembles the so-called backscatter data of inverse scattering. Such data arises if a single (infinitely small) pair of electrodes is used for driving currents and measuring voltage differences subsequently at various neighbouring locations on the boundary of the object of interest. We prove that this data uniquely determines an embedded simply connected and insulating (or ideally conducting) inclusion if the object has known constant background conductivity. This result is based on a link between the backscatter measurement and the Schwarzian derivative of a certain conformal mapping. Moreover, we present reconstructions of the so-called convex backscattering support for more general inhomogeneities.

Probing for inclusions in heat-conductive bodie

Patricia Gaitan, Hiroshi Isozaki, Olivier Poisson and Samuli Siltanen

University of Provence
France
poisson@latp.univ-mrs.fr

We consider the inverse boundary value problem arising from the equation of heat conduction. By using complex spherical waves, we give a mathematical theory and algorithm for probing the discontinuous part of the conductivity from the local boundary measurement without using knowledge of the initial data.

Exact shape reconstruction by one-step linearization in electrical impedance tomography

Bastian Harrach and Jin Keun Seo

Institute for Mathematics
University of Mainz
Staudingerweg 9, 55099 Mainz, Germany
harrach@mathematik.uni-mainz.de

For electrical impedance tomography (EIT), the linearized reconstruction method using the Fréchet derivative of the Neumann-to-Dirichlet map with respect to the conductivity has widely been used in the last three decades. However, little rigorous mathematical results are known regarding the errors caused by the linear approximation. In this talk we show that linearizing the inverse problem of EIT does not lead to shape errors for piecewise-analytic conductivities. If a solution of the linearized equations exists then it has the same outer support as the true conductivity change, no matter how large the latter is. Under an additional definiteness condition we also show how to approximately solve the linearized equation so that the outer support converges against the right one.

Approximation error approach for compensating modelling errors in diffuse optical tomography

Tanja Tarvainen, Ville Kolehmainen, Aki Pulkkinen, Marko Vauhkonen
Martin Schweiger, Simon R. Arridge and Jari P. Kaipio

Department of Physics
University of Kuopio,
P.O. Box 1627, 70211 Kuopio, Finland
Tanja.Tarvainen@uef.fi

Image reconstruction in diffuse optical tomography (DOT) is a non-linear ill-posed inverse problem. Thus, it tolerates measurement and modelling errors poorly. The modelling errors arise, for example, from using approximate forward models which are unable to describe the measurements with adequate accuracy. Furthermore, model reduction by using too coarse discretization in the solution of the forward problem can cause errors to the solution.

In DOT the most typical approach is to use the diffusion approximation (DA) to the radiative transfer equation (RTE) as forward model. The DA is basically a special case of the first order spherical harmonics approximation to the RTE, and thus it has some limitations. Firstly, the medium is assumed to be scattering dominated, and secondly light propagation can not be modelled accurately close to the collimated light sources and boundaries.

Recently, a Bayesian approach for the treatment of modelling errors has been proposed (Kaipio and Somersalo 2005 *Statistical and Computational Inverse Problems*). In this work, we consider utilising the approximation error approach for compensating modelling errors between the RTE and the DA. Further, different density discretizations of the forward solution are investigated.

A combined reconstruction-classification algorithm for diffuse optical tomography

Petri Hiltunen, Simon J.D. Prince and Simon Arridge

Department of Biomedical Engineering and Computational Science
Helsinki University of Technology
PO Box 2200, FI-02015 TKK, Finland
petri.hiltunen@tkk.fi

We have developed a new algorithm for diffuse optical tomography (DOT). DOT is a nonlinear and ill-posed problem. One way to regularise the DOT reconstruction is to assume that every pixel is independent and identically-distributed. Our method assume that pixels are from a mixture of Gaussian distribution, where mixture weights, mean, and variance are unknown. The developed algorithm can simultaneously reconstruct optical parameters, absorption and scattering coefficient, and gives probabilistic classification of the object.

The theory is written in a Bayesian framework and the algorithm can be described as an iterative variation between reconstruction step and estimation step until convergence. The reconstruction step, where optical parameters are estimated, is an optimization scheme with zeroth-order variable mean and variance Tikhonov-regularisation. The estimation step, where mixture model parameters are estimated, is an expectation-maximization algorithm.

We have shown using two dimensional simulated and three dimensional measured phantom data that this method enhances contrast and has good spatial accuracy.

The presented general algorithm has two interesting special cases: (i) background anatomy might be known from an other imaging modality, (ii) probabilities of background classes at every pixel might be known. The algorithm provides a framework for these cases and is applicable to other tomographic modalities.

Electrical impedance tomography imaging of concrete

Aku Seppänen, Kimmo Karhunen, Nuutti Hyvönen, Paulo J.M. Monteiro and Jari P. Kaipio

Department of Physics
University of Kuopio
Finland
aku.seppanen@uef.fi

We have tested the feasibility of Electrical Impedance Tomography (EIT) to non-destructive testing of concrete. Concrete is distinctly the most extensively used construction material in the world. There is a frequent need for reliable methods to evaluate the conditions of concrete structures – to assess the deterioration degree or the humidity state of concrete, etc.

Many of the currently used methods require breaking the material. Developing novel non-destructive methods is a topic of on-going research. Our recent findings have indicated that EIT holds potential for localizing cracks and reinforcing bars, as well as estimating the moisture distributions in concrete (see Karhunen et al.). In this presentation we show a number of examples of these experimental results. In addition, we consider a new computational approach to localization of reinforcing bars in concrete (Hyvönen et al.).

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Participants

Participants

Alessandrini, Giovanni	Università degli studi di Trieste	Italy
Arridge, Simon	University College London	United Kingdom
Auvinen, Harri	Lappeenranta University of Technology	Finland
Blåsten, Eemeli	University of Helsinki	Finland
Bosi, Roberta	University College London	United Kingdom
Bossuyt, Sven	Helsinki University of Technology	Finland
Bucataru, Ioan	Al.I.Cuza University of Iasi	Romania
Constantinescu, Oana	Al.I.Cuza University of Iasi	Romania
Cristofol, Michel	Université d'Aix-Marseille	France
Dahl, Matias	University College London	United Kingdom
Gaitan, Patricia	Aix-Marseille University	France
Gogie, Tsegaye Kassa	Bahir Dar University	Ethiopia
Haario, Heikki	Lappeenranta University of Technology	Finland
Harhanen, Lauri	Helsinki University of Technology	Finland
Harrach, Bastian	University of Mainz	Germany
Heikkinen, Jere	Lappeenranta University of Technology	Finland
Heino, Jenni	Helsinki University of Technology	Finland
Herman, Ben	Finnish Meteorological Institute	Finland
Hiltunen, Petri	Helsinki University of Technology	Finland
Hitrik, Michael	UCLA	USA
Hyvönen, Nuutti	Helsinki University of Technology	Finland
Johnson, Chris	University of Utah/SCI Institute	USA
Kaasalainen, Mikko	Tampere University of Technology	Finland
Kangaslampi, Riikka	Helsinki University of Technology	Finland
Karhunen, Kimmo	University of Kuopio	Finland
Krupchyk, Katya	University of Helsinki	Finland
Laine, Marko	Finnish Meteorological Institute	Finland
Lasanen, Sari	University of Oulu	Finland
Lassas, Matti	University of Helsinki	Finland
Lehikoinen, Anssi	Numcore Ltd/University of Kuopio	Finland
Lehtikangas, Ossi	University of Kuopio	Finland
Lehtinen, Markku	University of Oulu	Finland
Lesosky, Maia	Helsinki University of Technology	Finland
Markkanen, Markku	Eigenor Oy	Finland
Määttä, Matti	University of Helsinki	Finland
Niemi, Esa	University of Helsinki	Finland
Nissinen, Antti	University of Kuopio	Finland
Oksanen, Lauri	Helsinki University	Finland
Ola, Petri	University of Helsinki	Finland
Orispää, Mikko	Sodankylä Geophysical Observatory	Finland

Peltonen, Kirsi	Helsinki University of Technology	Finland
Perkkiö, Juha-Matti	Helsinki University of Technology	Finland
Piiroinen, Petteri	University of Helsinki	Finland
Pohjola, Valter	University of Helsinki	Finland
Poisson, Olivier	University of Provence	France
Pursiainen, Sampsa	Helsinki University of Technology	Finland
Päivärinta, Lassi	University of Helsinki	Finland
Roininen, Lassi	Sodankylä Geophysical Observatory	Finland
Saksman, Eero	University of Helsinki	Finland
Salmi, Max	Eigenor Oy	Finland
Sampo, Jouni	Lappeenranta University of Technology	Finland
Sandhu, Jan	University of Oulu	Finland
Seppänen, Aku	University of Kuopio	Finland
Serov, Valeriy	University of Oulu	Finland
Siltanen, Samuli	University of Helsinki	Finland
Simonaho, Simo-Pekka	University of Kuopio	Finland
Solonen, Antti	Lappeenranta University of Technology	Finland
Sundberg, Pauli	Eigenor Oy	Finland
Tarvainen, Tanja	University of Kuopio	Finland
Tietäväinen, Juha-Pekka	Helsinki University of Technology	Finland
Vauhkonen, Marko	University of Kuopio	Finland
Vesalainen, Esa	University of Helsinki	Finland
Vierinen, Juha	Sodankylä Geophysical Observatory	Finland
Zhou, Liang	Sodankylä Geophysical Observatory	Finland

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